Chapter 3. Error Estimates

➢ *3.1 Error ellipsoid*

There are three components that enter into the computation of the hypocentral error ellipsoid:

1. The estimated standard error of arrival times with zero weight code (SEZWC),
2. The weight code assigned to each arrival time, and
3. For each station, the partial derivatives of travel time with respect to latitude, longitude, and depth for the final hypocenter.

There are two options for assigning SEZWC, which is used to scale the ellipsoid. If TEST(29) (see 2.2.4) is positive, then SEZWC is reset for each event to be equal to the RMS residual. This has the disadvantage that the RMS may vary significantly from event to event and usually reflects more than simply random errors in the arrival-time readings. If there are very few readings, the RMS residual may be smaller than the true reading error; or the RMS may be larger due to systematic errors caused by an inappropriate velocity model. As an alternative, SEZWC will be fixed for all events to -TEST(29) if TEST(29) is negative. In this case the error ellipsoid will not reflect any systematic errors or blunders (very large, but rare arrival-time errors), but will give an indication of the relative error between any nearby events located with similar station distributions. If this latter option is used, the RMS residual of each event as well as the size of the error ellipsoid should be monitored for poor hypocentral solutions.

Large error-ellipsoid axes are often the result of partial derivatives with respect to one parameter that are all very small or all nearly equal. For example, for an earthquake near the center of a single ring of stations, the partial derivatives with respect to depth will be nearly the same for all of the stations. This leads to a trade off between depth and origin time because the partial derivative of travel time with respect to origin time is also the same for all stations (always equal to 1.0).

The semi-major principal axes of the 68% joint-confidence ellipsoid are output on the SUMMARY record for each earthquake. The printed output also includes two horizontal single 68% confidence estimates, the larger being called SEH, and the single variable 68 %-confidence estimate for depth, SEZ. The relationship of these error estimates to the error ellipsoid is shown in Figure 3-1. The relationship between a joint two-dimensional probability distribution (Pxy) and a one-dimensional distribution (Px) is illustrated in Figure 3-2. For each value of x, Px is equal to the integral over y of the joint-probability function Pxy. The ratio between s, the 68% confidence limit for x, and m, the maximum deviation of the 68% joint confidence ellipse in the x direction, is equal to the square root of the ratio of the 68% value of chi-square with one degree of freedom to the 68% value of chi-square with two degrees of freedom. Similarly, the scaling relationship between the shadow of the joint hypocentral ellipsoid and the joint-epicentral region is based on chi-square values for two and three degrees of freedom (Figure 3-3).

DEGREES OF FREEDOM	CHI SQUARE VALUE	SQUARE ROOT OF CHI SQUARE
1	1.00	1.00
2	2.30	1.52
3	3.51	1.87

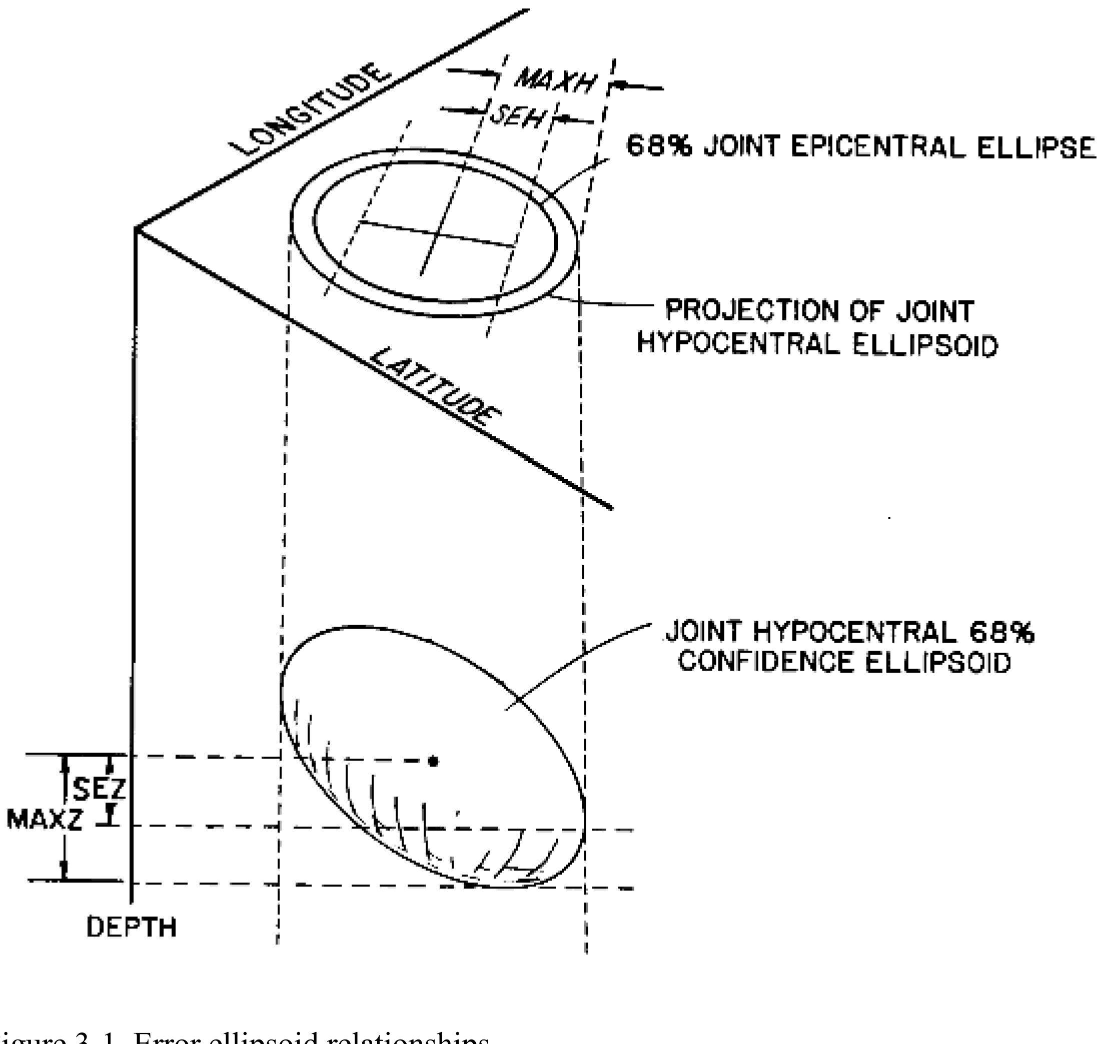

Figure 3-1. Error ellipsoid relationships.

SEH = MAXH/1.87.

SEZ = MAXZ/1.87

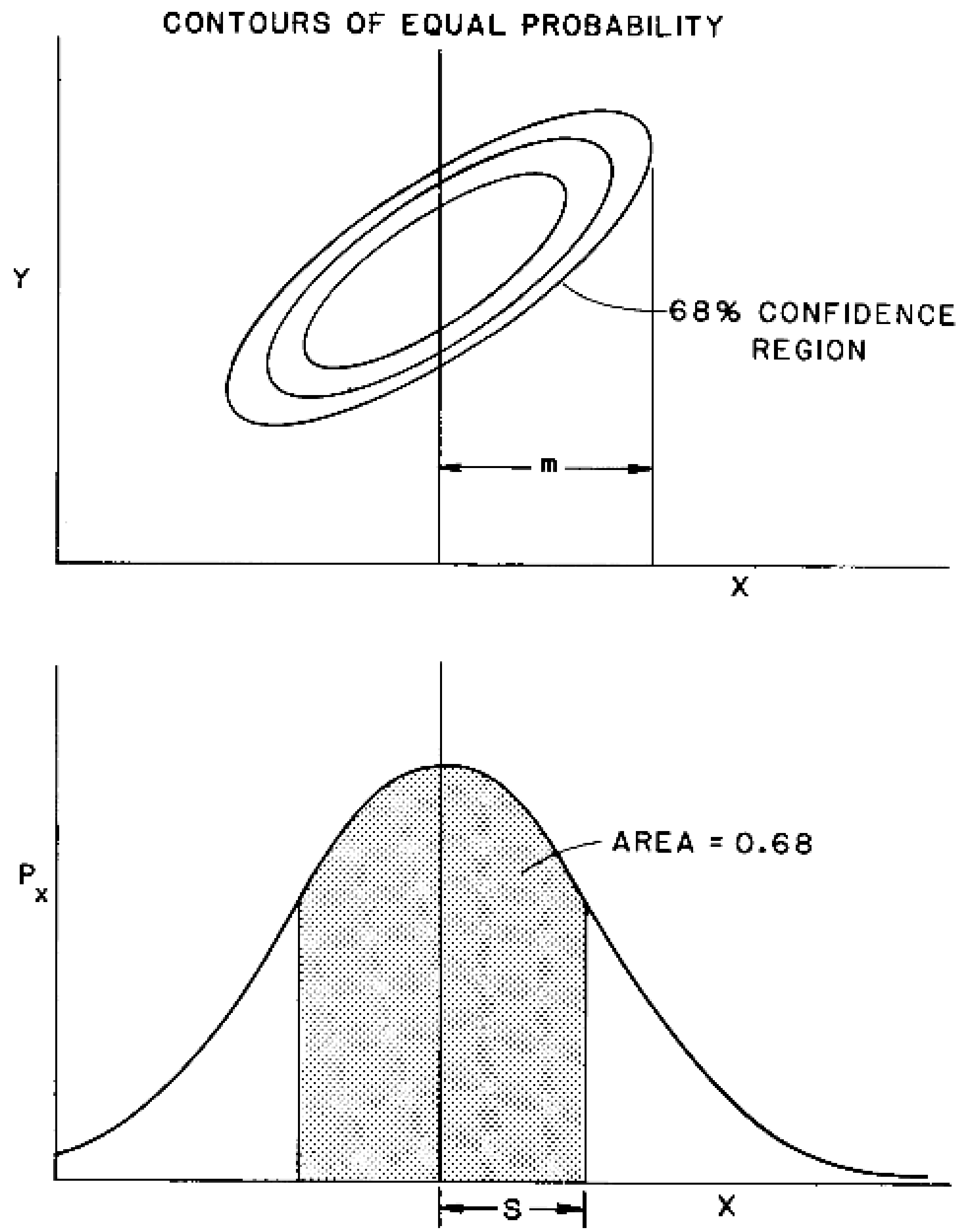

Figure 3-2.

Upper: Contours of equal probability in a two-dimensional probability distribution (Pxy).
Lower: One-dimensional probability distribution (Px) with same x scale as in upper figure.

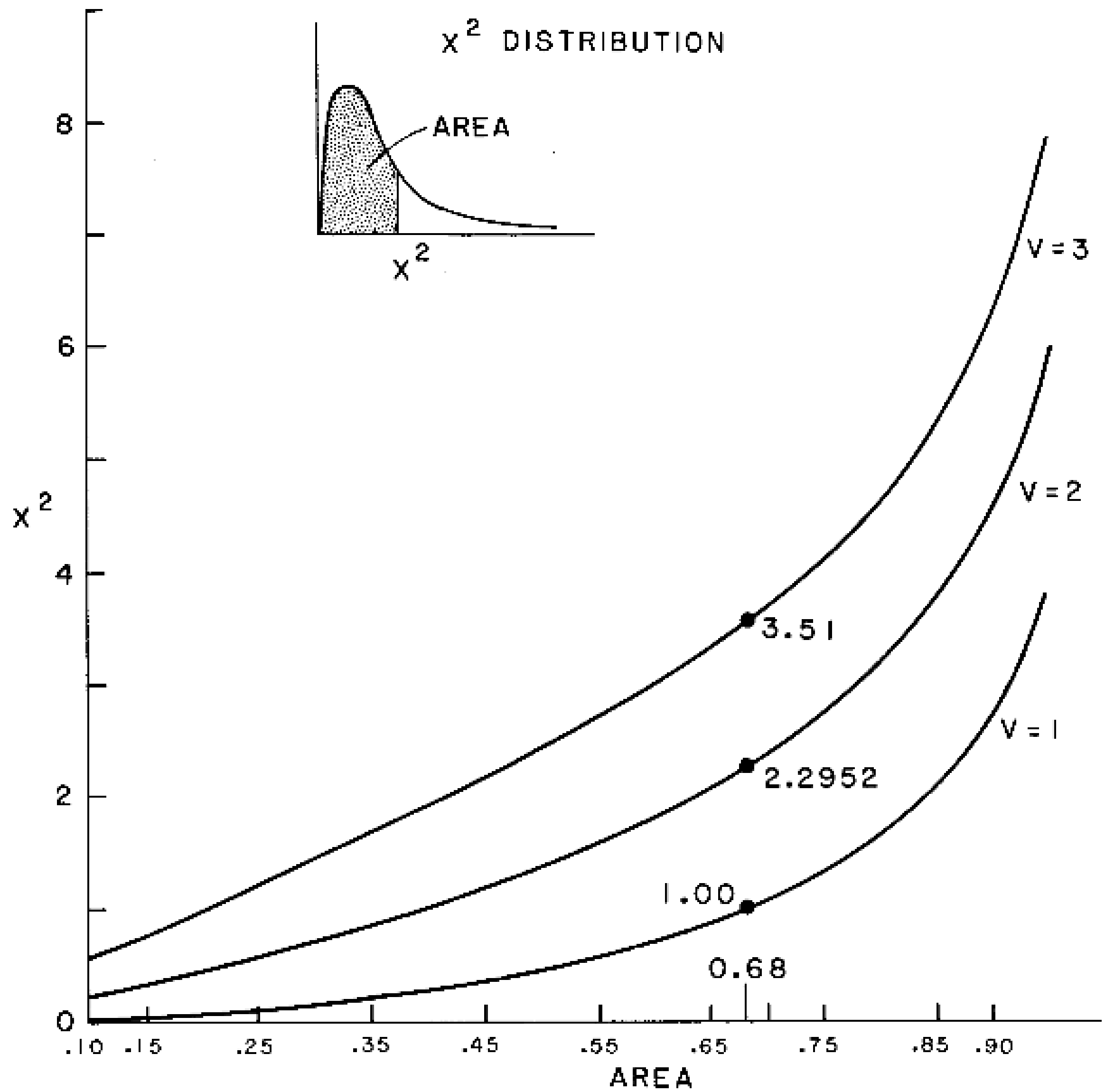

Figure 3-3. Chi-square versus area for 1, 2, and 3 degrees of freedom (V).

➢ *3.2 Global limits on depth*

The error ellipsoid is computed from the partial derivatives of travel times with respect to latitude, longitude and depth, evaluated at the final hypocenter determined for the earthquake. The travel times are not linear. Consequently, the error ellipsoid is an appropriate measure of the errors only to the extent that the partial derivatives are linear in the region nearby the final location and that there is only one spatial minimum of RMS residual. Earthquakes in southern Alaska often have a minimum in RMS residual at two different depths, and sometimes neither minimum is significantly lower than the other is. To help deal with these events, and also as a check on the error ellipsoid, the maximum upward and downward shifts of the depth that still have RMS < RMSLIM are computed and added to each SUMMARY record when the GLOBAL OPTION is used (See 2.2.3.11). RMSLIM is defined so that the depth limits correspond to one-standard-deviation in depth.

RMSLIM = SQRT(RMSZERO**2 + (YSE**2)/N)

where RMSZERO is the RMS residual of the final solution, YSE is the estimated standard error of the readings, and N is the number of P, S, and S-P observations used.

Nine events from southern Alaska are plotted in cross section in Figure 3-4. The final computed hypocenter, the projected error ellipsoid, and the depth limits computed with the GLOBAL OPTION are all shown. Note that the final hypocenter is not necessarily centered within the range of acceptable depths. In some cases this is due to the depth range spanning a local maximum. In others it is due to the iteration stopping because the minimum is essentially flat over a finite depth range. Also note that the error ellipsoid may indicate either a larger or smaller depth error than is indicated by the computed depth range. Although a vertical line segment indicates the depth range, the epicenter is not fixed during the search for alternative depths; so the true spatial pattern of alternative solutions is not indicated in this plot.

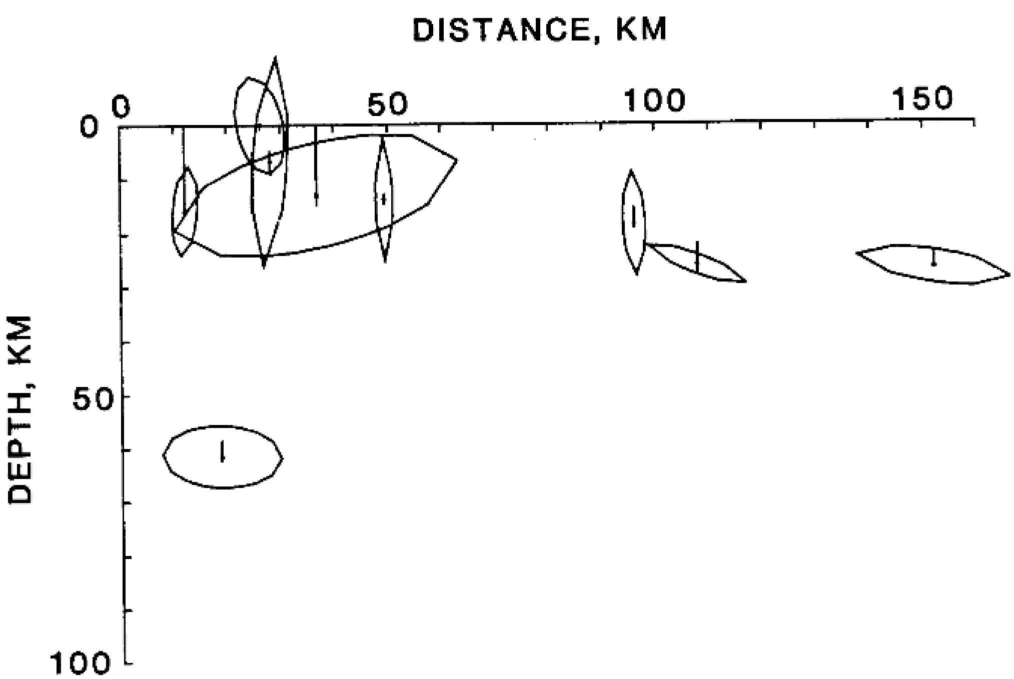

Figure 3-4. Cross section showing relationship between hypocenters, error ellipsoids and depth ranges computed with the GLOBAL OPTION.